SEISMIC OVERTURNING MOMENTS IN SINGLE-STOREY STRUCTURES WITH GROUND COMPLIANCE

by

J.H. Rainer (I)

SYNOPSIS

A method is presented for determining overturning moments in single-storey structures with flexible foundations subjected to earthquakes. By means of a transformation of frequency response curves, an equivalent single-degree-of-freedom model is derived to represent the overturning moment of the structure. This model is characterized by the fundamental resonance frequency of the interaction structure and an equivalent damping ratio. Response calculations are presented and response spectrum techniques are described for finding maximum overturning moments.

The results from a parameter study show that structures with a height-to-base width ratio greater than 1.0 generally have larger overturning moments than fixed-based structures with the same resonance frequency and interstorey damping.

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The subject of structure-ground interaction in buildings under seismic disturbances has lately received considerable attention (1, 2, 3, 4, 5, 6). These studies have concerned themselves exclusively with relative storey displacements in structures that rest on deformable foundations. Although the determination of interstorey displacements and the resulting forces are of great significance in the assessment of structural adequacy, the overturning moments associated with the dynamic forces are also of importance. The overturning moments affect the axial loads in the vertical supporting members of a building, which are then transmitted to the foundation where finally they have to be resisted by the soil.

If the axial load in the columns is exceeded, compression failures are possible, as was observed in some buildings after the Caracas 1967 earthquake (7). When the load capacity of the soil is exceeded, shear failures in the foundation materials may result, e.g. Niigata Earthquake 1964 (8). The behaviour of the foundation material is further complicated by tendency of some soils to liquefy under dynamic loading. In either case, a clear definition of the loads imposed by overturning moments is essential to be able to assess the safety and performance of structure and foundation under earthquake motions.

With the consideration of ground-structure interaction under dynamic loads, the determination of overturning moments becomes a more complex problem than for a comparable fixed-based structure. To aid in the understanding of the phenomenon involved, a relatively simple case is investigated first, namely that of the single-storey structure. The results presented here, however, will have direct applicability to structures such as elevated storage tanks and to structures that can be idealized realistically by the dynamic model of a simple oscillator founded on a flexible base.

The method of analysis employed here is to derive an equivalent single-degree-of-freedom (S. D. F.) model for the overturning moment of the interaction structure. This approach has been developed previously for studies of relative storey displacements in single-storey interaction structures under earthquake loadings (9). Among the advantages of using an equivalent S. D. F. model is the direct applicability of techniques that have been developed for S. D. F. oscillators, such as numerical integration and the response spectrum. Furthermore, extensive parameter studies can be performed without having to rely on the results of lengthy response calculations.

In the following pages are described the interaction model that was employed in the study, the derivation of an equivalent single-degree-of-freedom model to represent the overturning moment of the interaction structure, and numerical results from a parameter study. The use of the equivalent S. D. F. model is illustrated with specific response calculations and with earthquake response spectra. The results are discussed and general conclusions are presented.

INTERACTION MODEL

The interaction model under study is that shown in Fig. 1, which is identical to the one used in References 3 and 9. Since detailed derivations are available in these references, only the main results are presented here.

With the introduction of relative horizontal base displacement and rocking, the S. D. F. system has become a three-degree-of-freedom system. For the three generalized coordinates, relative interstorey displacement, horizontal displacement of structure, and rocking about the base, the equations of motion are, respectively,

$$m_1 \ddot{U}_H + c\dot{U}_m + kU_m = 0 \qquad (1)$$

$$m_1 \ddot{U}_H + m_o \ddot{U}_B + P = 0 \qquad (2)$$

$$I_1 \ddot{\Phi} + I_o \ddot{\Phi} + m_1 h \ddot{U}_H + M = 0 \qquad (3)$$

where

$$I_o = m_o \frac{r^2}{4} + m_1 \frac{r^2}{4},$$

$$I_1 = m_1 h^2, \; U_H = u_g + U_B + h\Phi + U_m,$$

and dots above a variable represent differentiation with respect to time. Of these, Eq. 3 is of primary interest for the determination of overturning moments. Under a sinusoidal base disturbance, $u_g = We^{ipt}$, the amplification factors X, Y and Z for relative storey displacement U_m, base displacement U_B and the angular rocking component Φ are given by

$$U_B = We^{ipt} X = u_g (X_1 + iX_2) \qquad (4)$$

$$U_m = We^{ipt} Z = u_g (Z_1 + iZ_2) \qquad (5)$$

$$\Phi = We^{ipt} Y = u_g (Y_1 + iY_2) \qquad (6)$$

and the forces between the base and the half-space are

$$P = P_o e^{ipt} = u_g (X - 1) A \qquad (7)$$

$$M = M_o e^{ipt} = u_g YB. \qquad (8)$$

For a circular base

$$A = Gr(K_H + iaC_H) \tag{9}$$

$$B = Gr^3(K_R + iaC_R) \tag{10}$$

where

$$K_H = \frac{f_{1H}}{(f_{1H})^2 + (f_{2H})^2}, \quad C_H = \frac{-\frac{f_{2H}}{a}}{(f_{1H})^2 + (f_{2H})^2},$$

$$K_R = \frac{f_{1R}}{(f_{1R})^2 + (f_{2R})^2}, \quad C_R = \frac{-\frac{f_{2R}}{a}}{(f_{1R})^2 + (f_{2R})^2};$$

the K's can be interpreted as S. D. F. stiffness and the C's as damping terms for the foundation (10) and are plotted in Fig. 2. These in turn are a function of the Bycroft coefficients f_1 and f_2 (11). The subscripts H and R refer to horizontal and rocking displacements, respectively. G = shear modulus of the ground, r = radius of base, $i = \sqrt{-1}$, $a = pr/V_s$ = non-dimensional frequency, V_s = shear wave velocity of ground, p = frequency. Substitution in the equation of motion, simplification and re-arrangement gives the following relationship for the steady state displacement amplification factor:

$$
\begin{pmatrix}
\left(1-\frac{\omega_o^2}{p^2}\right) & -\frac{2\lambda\omega_o}{p} & 1 & 0 & 1 & 0 \\
-\frac{2\lambda\omega_o}{p} & -\left(1-\frac{\omega_o^2}{p^2}\right) & 0 & -1 & 0 & -1 \\
1 & 0 & \left(1+\frac{1}{\alpha}-\frac{\omega_H^2}{p^2}\right) & -\frac{2\lambda_H\omega_H}{p} & 1 & 0 \\
0 & -1 & -\frac{2\lambda_H\omega_H}{p} & -\left(1+\frac{1}{\alpha}-\frac{\omega_H^2}{p^2}\right) & 0 & -1 \\
1 & 0 & 1 & 0 & \left(1+\frac{1+\alpha}{4\eta}-\frac{\omega_R^2}{p^2}\right) & -\frac{2\lambda_R\omega_R}{p} \\
0 & -1 & 0 & -1 & -\frac{2\lambda_R\omega_R}{p} & -\left(1+\frac{1+\alpha}{4\eta}-\frac{\omega_R^2}{p^2}\right)
\end{pmatrix}
\begin{pmatrix} Z_1 \\ Z_2 \\ X_1 \\ X_2 \\ hY_1 \\ hY_2 \end{pmatrix}
=
\begin{pmatrix} 0 \\ 0 \\ \frac{\omega_H^2}{p^2} \\ \frac{2\lambda_H\omega_H}{p} \\ 0 \\ 0 \end{pmatrix}
\qquad (11)
$$

which can be expressed more compactly as

$$\lfloor D \rceil \{d\} = \{f\} \qquad (11a)$$

In Eq. (11)

$$\frac{\omega_H^2}{p^2} = \frac{K_H}{a^2\beta}, \quad \frac{\omega_R^2}{p^2} = \frac{K_R}{a^2\beta\eta}$$

$$\lambda_H = \frac{C_H}{2(\beta K_H)^{\frac{1}{2}}}, \quad \lambda_R = \frac{C_R}{2(\beta\eta K_R)^{\frac{1}{2}}}$$

and

$$\alpha = \frac{m_o}{m_1}, \quad \beta = \frac{m_1}{\rho r^3}, \quad \eta = \left(\frac{h}{r}\right)^2, \quad \omega_o^2 = \frac{k}{m_1}$$

λ = relative inter-storey damping ratio,

ρ = density of ground

The transfer function for the system, $T_{u_g}^{d}$, due to ground displacement is obtained by rearranging Eq. 11.

$$T_{u_g}^{d} = \{d\} = [D]^{-1} \{f\} \qquad (12)$$

The transfer function for ground acceleration may then be obtained as follows:

$$T_{\ddot{u}_g}^{d} = \frac{1}{(ip)^2} T_{u_g}^{d} = \frac{-1}{p^2} T_{u_g}^{d} \qquad (13)$$

where d denotes the generalized displacement vector for the structure.

OVERTURNING MOMENTS

The overturning moment M acting on the elastic half-space as a result of the dynamic response of the structure is obtained from Eq. 3:

$$M = -\left[I_o\ddot{\Phi} + I_1\ddot{\Phi} + m_1 h\ddot{U}_H\right] \qquad (14)$$

Upon substitution of steady-state amplification factors, Eqs. 4 to 6, and appropriate structural constants, the transfer function for overturning moment relative to ground acceleration, $T_{\ddot{u}_g}^{M}$, is given by

$$T_{\ddot{u}_g}^{M} = \frac{M}{m_1 h\ddot{u}_g} = -\left[\left[1 + \frac{1+\alpha}{4\eta}\right] hY + X + Z\right] \qquad (15)$$

For a rigidly based structure, $Y = 0$, $X = 1$; thus

$$\frac{M}{m_1 h \ddot{u}_g} = -[1 + Z]. \quad (16)$$

X, Y, and Z may be computed from Eq. 12, and the addition in Eq. 15 has to be carried out with due regard to signs and real and imaginary components. The frequency response curve for the overturning moment of a typical structure (Structure No. 1, Table 1) is presented in Fig. 3 (a).

EQUIVALENT S. D. F. FOR OVERTURNING MOMENTS

The response of a linear dynamical system is completely determined by its frequency response curve, which represents a plot of the transfer function of the system versus frequency. The transfer function of overturning moment due to ground acceleration is given by Eq. 15, which, when plotted as a function of frequency, results in the frequency response curve for overturning moments shown in Fig. 3 (a). Also shown in dotted lines, is the frequency response curve for overturning moments of an S. D. F. system with the same resonance frequency Ω when subjected to ground acceleration $\ddot{u}_g$.

DERIVATION OF EQUIVALENT S. D. F. MODEL

To set up an equivalent S. D. F. model for overturning moments, it is necessary to convert the frequency response curve for overturning moments shown in Fig. 3 into that of an S. D. F. system. With reference to Fig. 3 (b), three conditions are to be satisfied by the equivalent S. D. F. model and the original interaction system: (1) identical resonance frequency; (2) agreement of ordinates of the frequency response curves away from resonance; and (3) agreement of ordinates of the frequency response curves at the resonance frequency.

1. Identical Resonance Frequency

Since the resonance frequency of the equivalent S. D. F. model is the same as the fundamental frequency of the interaction system, standard eigenvalue methods applied to the mathematical model of the interaction system will yield the required resonance frequency. By eliminating the rows and columns corresponding to the imaginary terms of the matrix in Eq. 11, the roots of the resulting determinant are the eigenvalues of the interaction system. The lowest frequency corresponds to the fundamental, and consequently to the resonance frequency of the equivalent S. D. F. model. Alternatively, a numerical search for the lowest resonance peak in the frequency response curves, using Eq. 11, will also give the resonance frequency of the equivalent S. D. F. model.

2. Agreement of Ordinates Away From Resonance

For an S. D. F. oscillator the quantity that is usually computed is the relative displacement between the base and the spring-supported mass. In order that spectral techniques and S. D. F. numerical integration methods may be employed for response calculation of interaction structures, the frequency response curve for overturning moments of the interaction system must be converted into the frequency response curve for relative displacement of the equivalent S. D. F. system.

The zero frequency intercept of the frequency response curve for relative displacement U_m, obtained by evaluating the transfer function $T_{\ddot{u}_g}^{U_m}$ from Eqs. 11, 12 and 13 at p = 0, corresponds to the displacement resulting from a base input of zero frequency, i. e. a constant acceleration $\ddot{u}_g = 1.0$. The resulting relative displacement of the S. D. F. system is then

$$U_m = \frac{m\ddot{u}_g}{k} = \frac{1}{\Omega^2}\ \ddot{u}_g = \frac{1}{\Omega^2} \qquad (17)$$

On the other hand, the zero frequency intercept for overturning moments is obtained from Eq. 15, by setting p = 0, Y = Z = 0 and X = 1, which then becomes

$$\left[\frac{M}{m_1 h \ddot{u}_g}\right]_{p=0} = -1.0$$

The ordinates of the frequency response curves for overturning moment at frequencies below resonance are brought to coincide with the ordinates of an S. D. F. system if the former are multiplied by $1/\Omega^2$. As will be explained later, agreement of ordinates above resonance is not completely satisfied by this multiplication factor due to differences in phase.

3. Agreement of Ordinates at Resonance

The amplitude of the resonance peak of the equivalent S. D. F. frequency response curve obtained by means of the above multiplication is now characterized by an equivalent damping ratio.

The damping ratio of an S. D. F. system can be computed from the relationship

$$\lambda = \frac{1}{2\xi} \qquad (18)$$

where ξ = nondimensional amplification factor of an S. D. F. oscillator at the resonance frequency Ω. The frequency response curve of $T_{\ddot{u}_g}^{U_m}$ of the equivalent S. D. F. model has to be converted to the nondimensional form $T_{u_g}^{U_m}$ in order that Eq. 18 may be applicable. As a corollary to Eq. 13, $T_{\ddot{u}_g}^{U_m}$ is to be multiplied by the variable p^2 to give $T_{u_g}^{U_m}$. But at the resonance frequency, p equals Ω. Therefore

$$\left[T_{u_g}^{U_m}\right]_{p=\Omega} = \Omega^2 \left[T_{\ddot{u}_g}^{U_m}\right]_{p=\Omega} \qquad (19)$$

It should also be recalled from preceding paragraphs that the constant multiplication factor $1/\Omega^2$ is applied to $T_{\ddot{u}_g}^{U_m}$ to achieve agreement with $T_{\ddot{u}_g}^{U_m}$ of the equivalent S. D. F. model at the zero frequency intercept. Consequently the amplitude ξ_e of the nondimensional amplification factor of the equivalent S. D. F. model at resonance becomes

$$\xi_e = \Omega^2 \left[\frac{1}{\Omega^2} \left[T_{\ddot{u}_g}^{M} \right]_{p = \Omega} \right]$$

$$\xi_e = \left[T_{\ddot{u}_g}^{M} \right]_{p = \Omega} \tag{20}$$

This means that for the calculation of the equivalent damping ratio λ_e, the actual amplitudes of the overturning moment transfer function at resonance, as given by Eq. 15, can be used in Eq. 18 (II).

With resonance frequency Ω, the multiplication factor $1/\Omega^2$, and the equivalent damping ratio λ_e, the frequency response curve of the equivalent S. D. F. system is completely described. Since all amplitudes of the frequency response curve for overturning moments have been multiplied by $1/\Omega^2$ any response computed with this equivalent S. D. F. model has to be multiplied by Ω^2 in order to obtain the overturning moment $M/m_1 h$.

RESPONSE CALCULATIONS FOR OVERTURNING MOMENTS

Figure 4 shows response calculations for overturning moments for Structures No. 1 and 2 of Table 1. For both interaction systems the overturning moment, shown by solid lines, is larger than for a rigidly based structure of the same natural frequency, as represented by the dotted curve. The responses obtained for Structure No. 1 from the equivalent S. D. F. model differ slightly from the "exact" one obtained by Fast Fourier transform (12) with transfer function of Eq. 24, for the following reason.

The transfer function for overturning moment, Eq. 15, is seen to be a composite quantity of all three degrees of freedom. Above the resonance frequency the amplitudes of the frequency response curve for overturning moment will be diminished by the relative base displacement, since it has phase opposite to that of rocking and relative displacement (Fig. 5). Consequently, for frequencies above resonance the frequency response curve of the equivalent S. D. F. model will not have the same proportion of amplitudes as the overturning moment frequency response curve. Some difference of response can thus be expected from the equivalent S. D. F. model as compared to the "exact" method using the transfer function and Fourier transform method. Since for most structures the relative base displacement is small compared to the interstorey and rocking displacements, the error will be small. For Structure No. 1, the discrepancy was in the order of 6% of the peak response amplitude.

II In reference 9 it has been demonstrated that under certain simplifying assumptions the equivalent damping λ_e for relative displacement and overturning moment are the same.

MAXIMUM OVERTURNING MOMENT FROM RESPONSE SPECTRA

Determination of the maximum overturning moment under a particular base motion may be achieved directly from the response spectrum for that particular earthquake. In accordance with the derivation of the equivalent S. D. F. system described above, the following procedure is indicated:

(1) The fundamental resonant frequency, ω_1, is determined, and the amplitudes of the overturning moment response curve are multiplied by $1/\omega_1^2$.

(2) The equivalent damping λ_e is determined after obtaining $M/m_1 h$ from an evaluation of Eq. 15 at resonance, or a parameter study such as is presented later.

(3) Corresponding to the frequency ω_1, the value of maximum relative displacement is read from the response spectrum and multiplied by ω_1^2 to obtain the value of the ratio for overturning moment, $M/m_1 h$.

An approximation to the maximum overturning moments of tall structures is obtained by taking the undamped spectral displacement S_D and multiplying it by ω_1^2 to get $M/m_1 h$; this gives a conservative estimate.

PARAMETER STUDY

Besides the great generality inherent in the method of derivation of the equivalent S. D. F. model, a parameter study can be conducted for a wide range of parameters without having to perform lengthy response calculations. The results obtained are presented so that resonance frequencies and equivalent damping can be found for structures having the specific parameters considered. In addition it is possible to arrive at some general conclusions regarding the behaviour of structures founded on flexible foundations.

The range of structural and foundation parameters considered is shown in Table 2. Parameter Set A includes elevated structures such as water tanks; Parameter Set B would include nuclear reactor containment vessels and other massive structures. It should be pointed out, however, that not all combinations of parameters presented may represent physically realizable systems.

PARAMETER SET A

For Parameter Set A of Table 2, when the ratio of fixed-based frequency to rocking frequency, ω_0/ω_Φ, is plotted against the ratio of fixed-based frequency to the fundamental resonance frequency, ω_1/ω_0, the values fall between the bounds shown in Fig. 6. The lower curve corresponds to the theoretical relationship derived by Merritt and Housner (1) for a flexible structure on a base with a rotational degree of freedom only:

$$\frac{\omega_o}{\omega_1} = \left[1 + \left[\frac{\omega_o}{\omega_\Phi}\right]^2 \right]^{\frac{1}{2}} \qquad (21)$$

For Parameter Set A of Table 2, the variation of the ratio M_S/M_I of peak magnitudes of the interaction system to that of a fixed-based S. D. F. system with the same natural frequency is shown in Figs. 7 and 8. It may be observed that for the tall narrow-based structures the peak for the interaction system is considerably larger than that of a fixed-based structure with the same natural frequency. Only for high values of a_o and relatively low structures are the overturning moment resonance peaks for interaction structures smaller than the S. D. F. case.

PARAMETER SET B

Similarly as for Parameter Set A, a plot of the ratios ω_o/ω_Φ versus ω_o/ω_1 for Parameter Set B of Table 2 gives the curves shown in Fig. 9. Again the relationship for a rocking base is indicated by broken lines. It may be observed that as the structures become taller, the frequency reduction approaches that of the flexible structure on a rocking base, Eq. 21.

In the study of resonance peak amplitudes, for Parameter Set B, Table 2, a similar phenomenon as for Parameter Set A is displayed. In Figures 10 and 11 for the structures with the larger ratio of height to base radius h/r, the resonance peaks of the overturning moment exceed those of the S. D. F. (i. e. the fixed-based S. D. F.), whereas for the low structure the peaks are smaller than for the S. D. F. Figure 10 shows that the peak overturning moment amplitudes for the structure with base mass m_o = 100 000 lb sec^2 in., shown by broken lines, are only slightly less than those for m_o= 400 000 lb sec^2/in., both top masses m_1 equal to 400 000 lb sec^2/in. and all other parameters being kept constant.

DISCUSSION OF RESULTS

From the results presented in the parameter study it may be observed that the resonance peaks of overturning moments for structures whose heights exceed the base diameter generally are larger than the resonance peaks of the fixed-based, S. D. F. system. As the frequency response curves for the interaction structure and the S. D. F. system both have the same zero frequency intercept, and are almost identical everywhere except near the resonance frequency, a comparison of resonance peaks then enables one to make qualitative generalizations regarding the response to an arbitrary input. That is, when the resonance peaks exceed those of the S. D. F. system, the response of the interaction structure will be larger or at most equal to that of the S. D. F. system. Another way of interpreting these results is by way of the damping ratio. A system with a larger resonance peak has a smaller damping ratio, and consequently a larger response under random type or steady-state input (III). It may therefore be deduced from Figs. 7, 8, 10 and 11 that for some structures with foundation interaction and a height exceeding the base diameter the overturning moment under earthquake motions will be larger than for a rigidly based S. D. F. structure with the same natural frequency and interstorey damping.

From results presented and the underlying derivation of the equivalent S. D. F. model, some significant parameters can be identified. These are: (1) the frequency reduction from ω_o to ω_1, (2) the non-dimensional frequency $a_o = \omega_o r/V_s$, and (3) the height and top mass of the structure. These three groups of parameters are not independent, but they provide a conveient set for the presentation and interpretation of results, as well as for their subsequent use in the response calculations using the equivalent S. D. F. system. From Figs. 9 and 10 it may be seen that the amplitude of resonance peaks as well as the frequency reductions are almost independent of the magnitude of the base mass.

The frequency reduction of all interaction systems investigated is primarily dependent on the ratio of rocking frequency to the fixed-based natural frequency of the structure, ω_Φ/ω_o, as may be seen from Figs. 6 and 9. But the rocking frequency in turn is directly proportional to the shear wave velocity of the ground, V_s. From this observation and the results of the parameter study in Figs. 7, 8, 10 and 11, it may be concluded that for reasonably tall structures the same amplification of overturning moments for a structure-foundation system is obtained as long as ω_o/V_s is constant. Because of this property, results presented are valid beyond the range of specific values of ω_o and V_s used, provided ω_o/V_s does not exceed its range considered here.

SUMMARY AND CONCLUSIONS

The method of analysis of the equivalent S. D. F. model has been applied to an investigation of overturning moments in single-storey structure-ground interaction systems. The equivalent S. D. F. model is obtained by matching the frequency response curve for overturning moments with that of an S. D. F. system and requires the determination of the resonance frequency, a multiplication factor and an equivalent damping ratio. This method permits an extensive parameter study and the isolation of the significant parameters for single-storey structures with foundation compliance. From the results presented, maximum overturning moments under arbitrary base motions can then be found by numerical integration or response spectrum techniques.

III It should be noted that this generalization is not valid for the comparison of resonance peaks for relative displacement, since the ordinates of the frequency response curves of the interaction system and the S. D. F. away from resonance are not of comparable magnitudes.

From a parameter study it is found that overturning moments in single-storey structures with ground compliance and with a height greater than base diameter generally exceed the corresponding value for a fixed-based S. D. F. system with the same natural frequency and inter-storey damping. This amplification effect is greater with larger height to base-width ratios.

The parameter study and the method of presentation of the results have indicated that the ratio of shear wave velocity to the natural frequency of the fixed-based structure is one of the important parameters in the structure-ground interaction phenomenon.

Although the results presented are limited to a certain class of relatively simple structures, they indicate that overturning moments should be given careful consideration in slender structures founded on elastic bases.

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TABLE 1

PARAMETERS FOR SAMPLE CALCULATIONS

Parameter	Unit	Structure No. 1	Structure No. 2
(a) Structural Parameters			
m_1	lb sec^2/in.	1000	4000
m_o	lb sec^2/in.	1000	1000
h	ft	40	80
r	ft	15	15
V_s	f.p.s.	300	800
ω_o	rad/sec	10	20
ω_1	rad/sec	7.59	7.60
λ	%	2.0	2.0
(b) Equivalent S.D.F. Model for Overturning Moment			
ω_1	rad/sec	7.59	7.60
ω_1^2		57.6	57.7
λ_e	%	1.39	0.20

TABLE 2

RANGES OF PARAMETERS

Variable	Parameter Set A	Parameter Set B
ω_o	5 to 20 rad/sec	5 to 20 rad/sec
m_1	1000 to 4000 lb sec^2/in.	100 000 and 400 000 lb sec^2/in.
m_o	1000 lb sec^2/in.	100 000 to 400 000 lb sec^2/in.
h	20 to 80 ft	40 to 160 ft
r	15 and 20 ft	60 ft
λ	2 per cent	2 per cent
V_s	300, 500 and 800 f.p.s.	500, 800 and 1600 f.p.s.

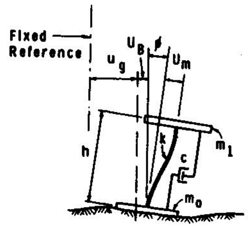


FIGURE 1 INTERACTION SYSTEM

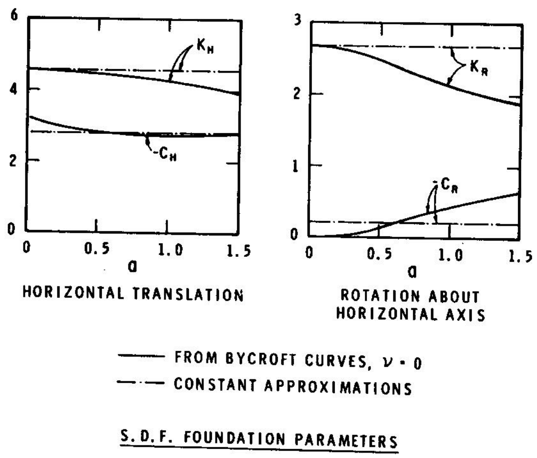


FIGURE 2

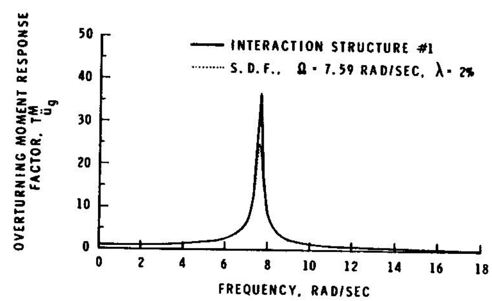


FIGURE 3(a) FREQUENCY RESPONSE CURVES FOR OVERTURNING MOMENTS

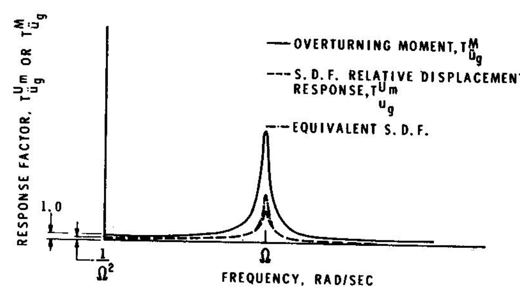


FIGURE 3 (b) FREQUENCY RESPONSE CURVES

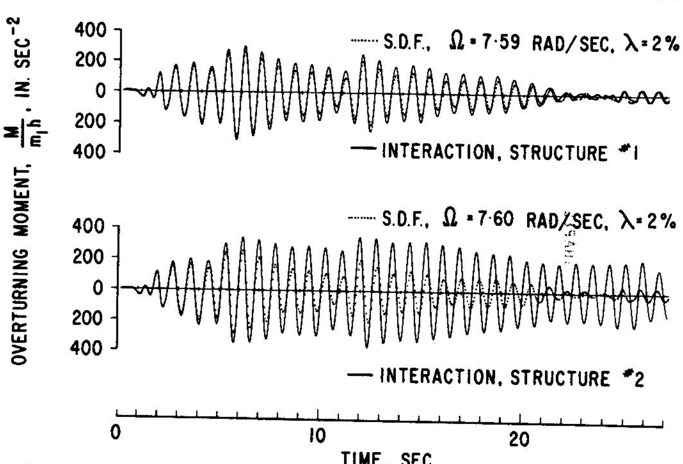
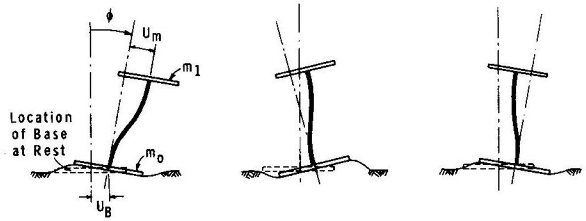


FIGURE 4

TIME HISTORY OF OVERTURNING MOMENTS. BASE MOTION: EL CENTRO 1940, N-S COMPONENT



a) Fundamental Mode   b) 2nd Mode   c) 3rd Mode

FIGURE 5

MODE SHAPES OF SINGLE-STOREY INTERACTION SYSTEM

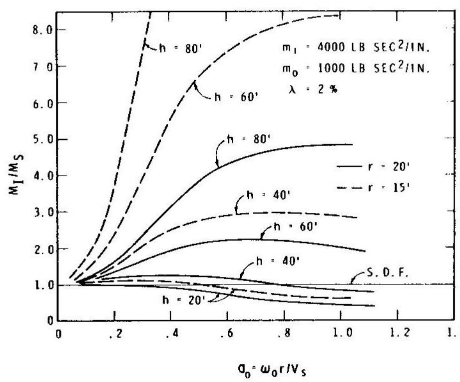


FIGURE 7
MAGNITUDES OF RESONANCE PEAKS FOR OVERTURNING MOMENTS, m_0 = 1000, m_1 = 4000 LB SEC²/IN., λ = 2%

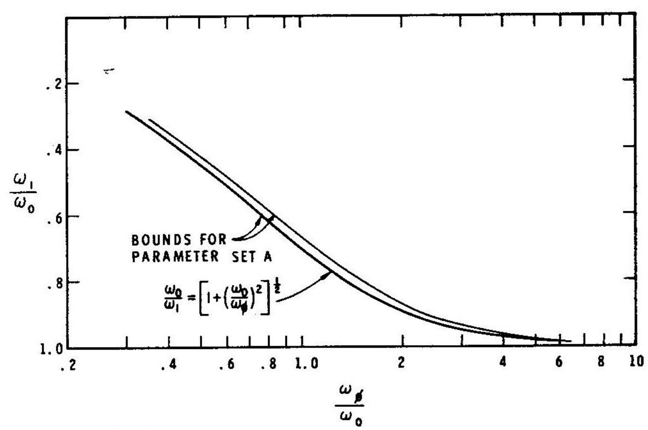


FIGURE 6

REDUCTION IN RESONANCE FREQUENCY FOR INTERACTION SYSTEMS, PARAMETER SET A

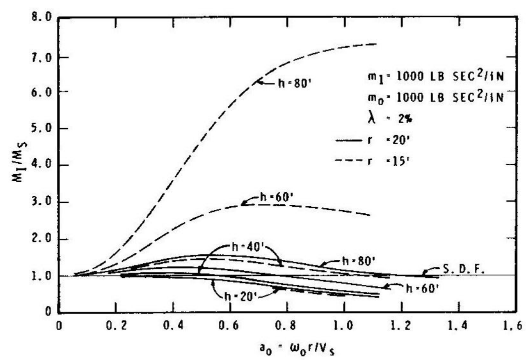


FIGURE 8

MAGNITUDES OF RESONANCE PEAKS FOR OVERTURNING MOMENTS, $m_0 = m_1$ = 1000 LB SEC²/IN, λ = 2%

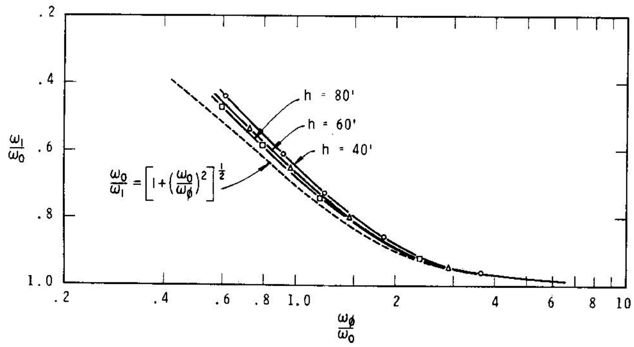


FIGURE 9

REDUCTION IN RESONANCE FREQUENCY FOR INTERACTION SYSTEMS, PARAMETER SET B

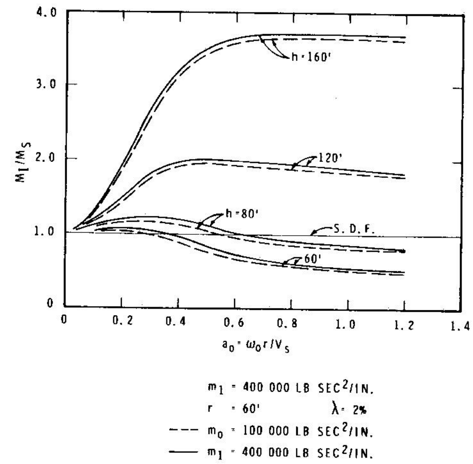


FIGURE 10

MAGNITUDES OF RESONANCE PEAKS FOR OVERTURNING MOMENTS, m_0 = 100 000 AND 400 000, m_I = 400 000 LB SEC^2/IN, λ = 2%

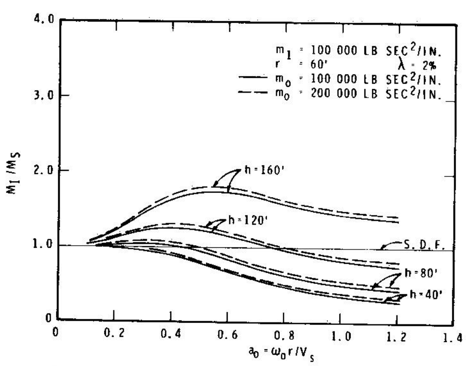


FIGURE 11

MAGNITUDES OF RESONANCE PEAKS FOR OVERTURNING MOMENTS, m_I = 100 000, m_0 = 100 000 AND 200 000 LB SEC^2/IN., λ = 2%

DISCUSSION OF PAPER NO. 10

SEISMIC OVERTURNING MOMENTS IN SINGLE STOREY STRUCTURES WITH GROUND COMPLIANCE

by

J. H. Rainer

Discussion by: M. Novak

The author presents a way in which Reissner's (Bycroft's) frequency functions f_1, f_2 can be introduced into the analysis of single storey structures to express the elastic and damping properties of the half-space (soil). This is a good approach because the half-space is taken into account without increasing the number of degrees of freedom.

The same procedure can be applied to systems with many degrees of freedom, e.g. Korenev, Baranov, 1964 and the effect of embedment can be also considered (paper no. 7).

As for quantitative applications to soils, the damping derived from the half-space should be considered with certain care. Experiments indicate that the theory tends to overestimate the damping in vertical direction and underestimate it with rocking if internal friction is neglected; layering can considerably reduce the geometric damping. Finally, the theory was developed for steady-state harmonic vibration not for nonstationary random motion as earthquakes are.

Reply by: J. H. Rainer

The basic structural model employed in this study has been used previously by Parmelle and Kobori, among others. The use of an equivalent single-degree-of-freedom model for overturning moment is thought to be novel, however. By determining the resonance frequency of the interaction system and an equivalent damping ratio from the magnitudes of resonance peaks, the response to non-stationary random motions such as an earthquake can then be found directly from a response spectrum. The technique presented, therefore, bridges the gap between the response for purely sinusoidal excitations and those for non-stationary motions such as earthquakes.

Numerous studies of multi-degree-of-freedom structures have been presented that incorporate the effects of flexible foundations. These have either been for sinusoidal excitations or numerical response calculations under random-type loads. As far as the author is aware, the ability to draw general conclusions from results obtained for sinusoidal disturbances which would be applicable to random-type inputs has not yet been achieved.

It is indeed possible to introduce the appropriate frequency functions f_1 and f_2 for any type of foundation behaviour, including the ones presented by the discusser in paper no. 7. One could also add additional amounts of damping to the elastic half-space results to account for small hysteretic energy loss in the soil. This is achieved by appropriate modifications of the C curves such as those presented in Figure 2. A limited study of this nature has been presented in Reference 9.

Question by: E. Varoglu

Could you comment on the reason for choosing a model with circular foundation?

In ground compliance analysis, the relation between the total force and the displacement depends upon the assumption about the stress distribution over the foundation area. To use a uniform stress distribution for a rigid foundation may not be adequate in obtaining total force, displacement relation for a rigid foundation.

Reply by: J.H. Rainer

The circular foundation was chosen as merely one example of a frequency dependent foundation. One could equally well choose a rectangular, square or other geometric shape as long as the experimental or theoretical complex stiffness characteristics are available. It should be pointed out that the results presented are valid for other foundation shapes provided their frequency-dependent characteristics resemble those given in Figure 2. This is the case for the square and rectangular foundations treated in Reference 10.

The writer agrees with the discusser that the stress distribution under the footing affects the stiffness characteristics. The work presented here, however, does not concern itself directly with this aspect. Rather, a statement of the problem treated might be as follows: given appropriate flexible foundation properties, what is the structural response of the interaction system under earthquake-type loadings? The suitability and applicability of a theoretical model to represent a particular foundation and its associated soil must then be considered for any specific application. In view of the results presented in Figs. 6 and 9, small variations in foundation stiffness should not greatly affect the frequency ratio ω_1/ω_0 and, as a consequence, the structural response of the interaction system.